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Plane Couette flow of binary gaseous mixture in the whole range of the Knudsen number

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Abstract

The Couette flow of binary gaseous mixtures is studied on the basis of the McCormack model of the Boltzmann equation, which was solved numerically by the discrete velocity method. The calculations were carried out for three mixtures of noble gases: neon–argon, helium–argon, and helium–xenon. The stress tensor and bulk velocity of both species were calculated for several values of the gas rarefaction in the range from 0.01 to 40 for three values of the molar concentrations: 0.1, 0.5 and 0.9. The numerical solution together with an analytical solution based on the slip boundary condition cover the whole range of the gas rarefaction. It was showed that the Couette flow is weakly affected by the intermolecular interaction law.

Keywords: Rarefied gases; Mixtures; Couette flow; Knudsen number

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1. Introduction

The plane Couette flow is a classical problem of rarefied gas dynamics, which was studied profoundly by many authors in case of a single gas, see, e.g. the works [1–7]. Recently, such kind of flows attracts a great interest especially of scientists and engineers working in the field of microfluidics and MEMS. The fact is that, in a small scale the damping force, i.e. the stress tensor between moving parts of MEMS, cannot be calculated applying the continuum mechanics equations but it must be calculated on the basis of the kinetic equations applying the methods of rarefied gas dynamics. Of course, the Couette flow is realized in many other practical situations, e.g. in drag pumps, where the gas is also so rarefied that the continuum mechanics is not valid any more.

In practice one deals with gaseous mixtures more frequently than with a single gas. In spite of the great practical importance there are very few papers in the open literature concerning the Couette flow of gaseous mixtures at intermediate Knudsen numbers. To our knowledge, just the papers by Onishi [8], by Valougeorgis [9] and by Siewert [10] reported some numerical results about this topic based on the Hamel model [11] of the Boltzmann equation [12]. However, this model is not complete in sense that it does not provide the correct expressions of all transport kinetic coefficients, i.e. viscosity, thermal conductivity, diffusion and thermal diffusion. To obtain more reliable results it is necessary to apply a model without this shortcoming, e.g. the model proposed by McCormack [13]. Besides the possibility to reproduce all transport coefficients the McCormack model allows us to use different laws of the intermolecular interaction. That is why this model is successfully used in practical

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calculations, see, e.g. [14–16]. A comparison of the results on the slip coefficients obtained from the McCormack model in Refs. [15–17] with those obtained from the exact Boltzmann equation [18–20] confirmed the reliability of this model equation. On the other hand, a numerical solution of model equations requires significantly less numerical efforts than a solution of the exact Boltzmann equation.

The aim of the present paper is to solve the Couette flow problem for a binary gaseous mixture in the whole range of the gas rarefaction applying the McCormack model [13].

2. Statement of the problem

Let us consider a binary gaseous mixture between two parallel plates fixed at $y' = \pm H/2$. The equilibrium state is perturbed by a motion of the plates in the x' direction with velocities $\pm U/2$, respectively. Thus, H is the distance between the plates and U is their relative velocity. We assume that the plate velocities are small compared with the characteristic molecular velocity of the mixture, i.e.

$$U \ll v_0, \quad v_0 = \left(\frac{2kT_0}{m}\right)^{1/2}, \tag{1}$$

where k is the Boltzmann constant, T_0 is an equilibrium temperature, m is the mean molecular mass

$$m = C_0 m_1 + (1 - C_0) m_2, \tag{2}$$

C_0 is a molar concentration in equilibrium

$$C_0 = \frac{n_{01}}{n_{01} + n_{02}}, \tag{3}$$

$n_{0\alpha}$ $(\alpha = 1, 2)$ is the equilibrium number density of species α, m_α is its molecular mass.

We are going to calculate the stress tensor P_{xy} between the plates and the mean velocities of every species $u'_{\alpha x}$ $(\alpha = 1, 2)$. The hydrodynamic velocity of the mixture u'_x related to $u'_{\alpha x}$ as

$$u'_x = \frac{1}{\varrho_1 + \varrho_2}(\varrho_1 u'_{1x} + \varrho_2 u'_{2x}), \quad \varrho_\alpha = n_{0\alpha} m_\alpha. \tag{4}$$

The problem will be solved in the whole range of the gas rarefaction δ defined as

$$\delta = \frac{H P_0}{\mu v_0}, \tag{5}$$

where μ is the mixture viscosity at the temperature T_0, and P_0 is an equilibrium pressure.

For further derivations it is convenient to introduce the following dimensionless quantities

$$\mathbf{c}_\alpha = \left(\frac{m_\alpha}{2kT_0}\right)^{1/2} \mathbf{v}_\alpha, \qquad u_\alpha = \frac{u'_{\alpha x}}{U}, \qquad u = \frac{u'_x}{U}, \tag{6}$$

$$\Pi = -\frac{v_0}{2U P_0} P_{xy}, \quad y = \frac{y'}{H}, \tag{7}$$

where $\mathbf{v}_\alpha$ is the molecular velocity of species α.

3. Hydrodynamic and slip flow regimes

In the hydrodynamic regime $(\delta \to \infty)$ the solution based on the Navier–Stokes equation with the non-slip boundary condition is trivial

$$u'_x = \frac{U}{H} y', \qquad P_{xy} = -\mu \frac{\partial u'_x}{\partial y'} = -\mu \frac{U}{H}. \tag{8}$$

In terms of the dimensionless quantities this solution reads

$$u = y, \qquad \Pi = \frac{1}{2\delta}. \tag{9}$$

A moderate gas rarefaction can be considered using the slip boundary conditions [21] at the plates

$$u'_x = \pm\frac{U}{2} \mp \sigma_{\mathrm{P}}\frac{\mu v_0}{P}\frac{\partial u'_x}{\partial y'}, \quad \text{at } y' = \pm\frac{H}{2}. \tag{10}$$

Then, instead of the solution (8) we obtain

$$u'_x = \frac{U}{H}\left(1 + \frac{2\sigma_{\mathrm{P}}}{\delta}\right)^{-1} y', \qquad P_{xy} = -\mu\frac{U}{H}\left(1 + \frac{2\sigma_{\mathrm{P}}}{\delta}\right)^{-1}. \tag{11}$$

Then the dimensionless solution reads

$$u = \left(1 + \frac{2\sigma_{\mathrm{P}}}{\delta}\right)^{-1} y, \qquad \Pi = \frac{1}{2\delta + 4\sigma_{\mathrm{P}}}. \tag{12}$$

Numerical data on the viscous slip coefficient σ_{P} can be found in [21] for a single gas and in [15,18,19,22] for gaseous mixtures.

4. Kinetic equation and boundary conditions

Although, the solution (12) can be used for a larger range of the gas rarefaction δ than Eq. (9), it continues to be restricted by large values of the gas rarefaction δ. To obtain the solution at an arbitrary gas rarefaction the system of the Boltzmann equations [12] must be solved. Since the plate velocities are small in accordance with (1) the Boltzmann equation can be linearized by the standard manner representing the distribution function of every species as

$$f_\alpha(\mathbf{r}, \mathbf{c}_\alpha) = f_\alpha^{\mathrm{M}}(\mathbf{c}_\alpha)\left[1 + h_\alpha(y, \mathbf{c}_\alpha)\frac{U}{v_0}\right], \quad \alpha = 1, 2, \tag{13}$$

where f_α^{M} is the Maxwellian distribution function corresponding to the equilibrium state

$$f_\alpha^{\mathrm{M}} = n_{0\alpha}\left(\frac{m_\alpha}{2\pi k T_0}\right)^{3/2}\exp\left(-c_\alpha^2\right), \tag{14}$$

$h_\alpha(y, \mathbf{c}_\alpha)$ are perturbation functions, which obey the two coupled Boltzmann equations [12]

$$c_{\alpha y}\frac{\partial h_\alpha}{\partial y} = H\left(\frac{m_\alpha}{2kT_0}\right)^{1/2}\sum_{\beta=1}^{2}\hat{L}_{\alpha\beta}h, \quad \alpha = 1, 2, \tag{15}$$

$\hat{L}_{\alpha\beta}$ is the linearized operator of collisions between species α and β. In the present work we apply the McCormack model kinetic equation [13], which for the problem in question reads

$$\begin{aligned}\hat{L}_{\alpha\beta}h = {} & -\gamma_{\alpha\beta}h_\alpha + 2\left(\frac{m_\alpha}{m}\right)^{1/2}\left[\gamma_{\alpha\beta}u_\alpha - \nu_{\alpha\beta}^{(1)}(u_\alpha - u_\beta) - \frac{\nu_{\alpha\beta}^{(2)}}{2}\left(q_\alpha - \frac{m_\alpha}{m_\beta}q_\beta\right)\right]c_{\alpha x} \\ & + 4\left[\left(\gamma_{\alpha\beta} - \nu_{\alpha\beta}^{(3)}\right)\Pi_\alpha + \nu_{\alpha\beta}^{(4)}\Pi_\beta\right]c_{\alpha x}c_{\alpha y} + \frac{4}{5}\left(\frac{m_\alpha}{m}\right)^{1/2}\left[\left(\gamma_{\alpha\beta} - \nu_{\alpha\beta}^{(5)}\right)q_\alpha + \nu_{\alpha\beta}^{(6)}\left(\frac{m_\beta}{m_\alpha}\right)^{1/2}q_\beta \right. \\ & \left. - \frac{5}{4}\nu_{\alpha\beta}^{(2)}(u_\alpha - u_\beta)\right]c_{\alpha x}\left(c_\alpha^2 - \frac{5}{2}\right), \quad \alpha, \beta = 1, 2,\end{aligned} \tag{16}$$

where the quantities $\nu_{\alpha\beta}^{(n)}$ are defined as

$$\nu_{\alpha\beta}^{(1)} = \frac{16}{3}\frac{m_{\alpha\beta}}{m_\alpha}n_\beta\Omega_{\alpha\beta}^{(11)}, \tag{17}$$

$$\nu_{\alpha\beta}^{(2)} = \frac{64}{15}\left(\frac{m_{\alpha\beta}}{m_\alpha}\right)^2 n_\beta\left(\Omega_{\alpha\beta}^{(12)} - \frac{5}{2}\Omega_{\alpha\beta}^{(11)}\right), \tag{18}$$

$$\nu_{\alpha\beta}^{(3)} = \frac{16}{5}\frac{m_{\alpha\beta}^2}{m_\alpha m_\beta}n_\beta\left(\frac{10}{3}\Omega_{\alpha\beta}^{(11)} + \frac{m_\beta}{m_\alpha}\Omega_{\alpha\beta}^{(22)}\right), \tag{19}$$

$$\nu_{\alpha\beta}^{(4)} = \frac{16}{5}\frac{m_{\alpha\beta}^2}{m_\alpha m_\beta}n_\beta\left(\frac{10}{3}\Omega_{\alpha\beta}^{(11)} - \Omega_{\alpha\beta}^{(22)}\right), \tag{20}$$

$$\nu_{\alpha\beta}^{(5)} = \frac{64}{15}\left(\frac{m_{\alpha\beta}}{m_\alpha}\right)^3 \frac{m_\alpha}{m_\beta} n_\beta \left[\Omega_{\alpha\beta}^{(22)} + \left(\frac{15}{4}\frac{m_\alpha}{m_\beta} + \frac{25}{8}\frac{m_\beta}{m_\alpha}\right)\Omega_{\alpha\beta}^{(11)} - \frac{1}{2}\frac{m_\beta}{m_\alpha}\left(5\Omega_{\alpha\beta}^{(12)} - \Omega_{\alpha\beta}^{(13)}\right)\right], \tag{21}$$

$$\nu_{\alpha\beta}^{(6)} = \frac{64}{15}\left(\frac{m_{\alpha\beta}}{m_\alpha}\right)^3 \left(\frac{m_\alpha}{m_\beta}\right)^{3/2} n_\beta \left(-\Omega_{\alpha\beta}^{(22)} + \frac{55}{8}\Omega_{\alpha\beta}^{(11)} - \frac{5}{2}\Omega_{\alpha\beta}^{(12)} + \frac{1}{2}\Omega_{\alpha\beta}^{(13)}\right), \tag{22}$$

$m_{\alpha\beta}$ is the reduced mass

$$m_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}, \tag{23}$$

$\Omega_{\alpha\beta}^{(ij)}$ are the omega integrals, which depend on the intermolecular interaction potential. For the hard sphere model they read

$$\Omega_{\alpha\beta}^{(ij)} = \frac{(j+1)!}{8}\left[1 - \frac{1+(-1)^i}{2(i+1)}\right]\left(\frac{\pi kT}{2m_{\alpha\beta}}\right)^{1/2}(d_\alpha + d_\beta)^2, \tag{24}$$

where d_α is the molecular diameter of species α.

The dimensionless moments of the distribution function are given by

$$u_\alpha(y) = \frac{1}{\pi^{3/2}}\left(\frac{m}{m_\alpha}\right)^{1/2} \int \exp\left(-c_\alpha^2\right) h_\alpha(y, \mathbf{c}_\alpha) c_{\alpha x}\, \mathrm{d}\mathbf{c}_\alpha, \tag{25}$$

$$q_\alpha(y) = \frac{1}{\pi^{3/2}}\left(\frac{m}{m_\alpha}\right)^{1/2} \int \exp\left(-c_\alpha^2\right) h_\alpha(y, \mathbf{c}_\alpha) c_{\alpha x}\left(c_\alpha^2 - \frac{5}{2}\right) \mathrm{d}\mathbf{c}_\alpha, \tag{26}$$

$$\Pi_\alpha(y) = \frac{1}{\pi^{3/2}} \int \exp\left(-c_\alpha^2\right) h_\alpha(y, \mathbf{c}_\alpha) c_{\alpha x} c_{\alpha y}\, \mathrm{d}\mathbf{c}_\alpha. \tag{27}$$

Note, the total reduced stress tensor Π defined by Eq. (7) is related to Π_α as

$$\Pi = -\left[C_0 \Pi_1 + (1 - C_0)\Pi_2\right]. \tag{28}$$

The parameters $\gamma_{\alpha\beta}$ are proportional to the collision frequency between species α and β and appear only in the combinations

$$\gamma_1 = \gamma_{11} + \gamma_{12}, \qquad \gamma_2 = \gamma_{21} + \gamma_{22}. \tag{29}$$

Thus, it is sufficient to define γ_1 and γ_2 for which we use the same expressions as in the previous works [14–17], i.e.

$$\gamma_\alpha = \frac{P_{0\alpha}}{\mu_\alpha}, \quad \alpha = 1, 2, \tag{30}$$

where $P_{0\alpha} = n_{0\alpha} k T_0$ is the equilibrium partial pressure and μ_α is the partial viscosity given as

$$\mu_\alpha = P_{0\alpha} \frac{\Psi_\beta + \nu_{\alpha\beta}^{(4)}}{\Psi_\alpha \Psi_\beta - \nu_{\alpha\beta}^{(4)} \nu_{\beta\alpha}^{(4)}}, \quad \Psi_\alpha = \nu_{\alpha\alpha}^{(3)} - \nu_{\alpha\alpha}^{(4)} + \nu_{\alpha\beta}^{(3)}, \tag{31}$$

where $\alpha = 1, 2$ and $\beta \neq \alpha$. The viscosity of the mixture μ is equal to the sum of the partial viscosities, i.e. $\mu = \mu_1 + \mu_2$. Such a choice of the quantities γ_1 and γ_2 guarantees that in the limit of a single gas the McCormack model (16) vanishes to the S model, which provides reliable results [21].

We assume the diffuse reflection that means

$$h_\alpha\left(\pm\frac{H}{2}, \mathbf{c}_\alpha\right) = \pm\left(\frac{m_\alpha}{m}\right)^{1/2} c_{\alpha x}, \quad \text{for } c_{\alpha y} \lessgtr 0. \tag{32}$$

In the free molecular regime ($\delta = 0$) the collision operator $\hat{L}_{\alpha\beta} h$ vanishes and Eq. (15) with the boundary condition (32) has the trivial solution, i.e.

$$h(y, \mathbf{c}_\alpha) = \pm\left(\frac{m_\alpha}{m}\right)^{1/2} c_{\alpha x} \quad \text{for } c_{\alpha y} \lessgtr 0. \tag{33}$$

Then, the reduced stress tensor is obtained with the help of Eqs. (27) and (28)

$$\Pi = \frac{C_0\sqrt{m_1} + (1 - C_0)\sqrt{m_2}}{2\sqrt{\pi m}}. \tag{34}$$

In the transition regime the kinetic equations (15) with (16) were solved numerically by the discrete velocity method [23] with the relative numerical error less than 0.1%. The numerical accuracy was estimated by comparing the results obtained for different grid parameters.

The intermolecular interaction law influences the solution of the kinetic equation (15) via the quantities $\nu_{\alpha\beta}^{(n)}$ defined by Eqs. (17)–(22), which contain the omega integrals. To investigate the influence of the intermolecular interaction law on the stress tensor two types of the potential were used: hard sphere model and realistic potential. When we use the hard sphere model we need just the ratios of the molecular diameters d_2/d_1, which are related to the ratios of the viscosities and the masses as

$$\frac{d_2}{d_1} = \left(\frac{\mu_1}{\mu_2}\right)^{1/2} \left(\frac{m_2}{m_1}\right)^{1/4}. \tag{35}$$

To calculate the ratios d_2/d_1 the experimental data [24] on the viscosity of every gas at the temperature $T = 300$ K were used. As a result the following values of d_2/d_1 were obtained: 1.406, 1.665 and 2.226 for the mixtures Ne–Ar, He–Ar and He–Xe, respectively.

The transport coefficients of mixtures, i.e. viscosity, thermal conductivity, diffusion coefficient, and thermal diffusion coefficient are related to the omega integrals [12,14,24]. Thus, the omega integrals can be calculated from the transport coefficients. Kestin et al. [24] obtained analytical expressions of the omega integrals, which provide experimental values of all transport coefficients and of the second virial coefficient for mixtures in an wide range of the temperature. In our calculations we used these expressions assuming the same temperature as that for the hard spheres, i.e. $T = 300$ K. Since the omega integrals based on the expressions by Kestin et al. [24] reproduce the transport coefficients we consider that these integrals correspond to a realistic potential of the intermolecular interaction.

5. Numerical results and discussions

The calculations were carried out for the three mixtures of noble gases: neon–argon (Ne–Ar), helium–argon (He–Ar), and helium–xenon (He–Xe). These mixtures have the quite different molecular mass ratios and allow us to investigate the dependence of the stress tensor on this parameter. The rarefaction parameter was varied in the range of δ from 0.01 to 40. Three values of the concentration were considered: $C_0 = 0.1$, 0.5 and 0.9. The numerical results on the stress tensor Π are given in Tables 1 and 2 for the realistic potential and hard sphere model, respectively. For two values of the rarefaction parameter $\delta = 1$ and 10 the dependence of the stress tensor on the concentration C_0 is shown in Figs. 1 and 2, respectively, in the case of the realistic potential. From these data we conclude:

(i) The reduced stress tensor Π always decreases by increasing the rarefaction parameter δ, that was expected since for a single gas the behavior of the stress tensor is the same.
(ii) At a fixed rarefaction parameter δ the reduced stress tensor Π for a mixture ($0 < C_0 < 1$) is always smaller than for a single gas ($C_0 = 0$ and $C_0 = 1$). A minimum value of the stress tensor is observed at $C_0 = 0.85$. Note, in accordance with Eq. (7)

Table 1
Stress tensor Π vs the rarefaction parameter δ and concentration C_0, realistic potential

δ	Π								
	Ne–Ar			He–Ar			He–Xe		
	$C_0 = 0.1$	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
0.0	0.2810	0.2781	0.2802	0.2755	0.2504	0.2490	0.2723	0.2308	0.2032
0.01	0.2785	0.2758	0.2778	0.2732	0.2484	0.2471	0.2701	0.2291	0.2019
0.02	0.2762	0.2735	0.2755	0.2709	0.2466	0.2452	0.2679	0.2275	0.2007
0.04	0.2719	0.2691	0.2711	0.2667	0.2431	0.2418	0.2638	0.2245	0.1983
0.1	0.2601	0.2576	0.2594	0.2554	0.2336	0.2324	0.2527	0.2162	0.1919
0.2	0.2437	0.2414	0.2431	0.2396	0.2201	0.2191	0.2371	0.2045	0.1826
0.4	0.2180	0.2160	0.2175	0.2147	0.1987	0.1979	0.2126	0.1856	0.1674
1.0	0.1688	0.1675	0.1685	0.1668	0.1566	0.1562	0.1654	0.1480	0.1361
2.0	0.1247	0.1239	0.1246	0.1236	0.1177	0.1175	0.1228	0.1125	0.1054
4.0	0.08281	0.08242	0.08274	0.08229	0.07957	0.07949	0.0819	0.07700	0.07358
10.0	0.04150	0.04139	0.04148	0.04136	0.04065	0.04063	0.04126	0.03994	0.03899
20.0	0.02268	0.02264	0.02267	0.02264	0.02242	0.02241	0.02261	0.02220	0.02190
40.0	0.01189	0.01188	0.01189	0.01188	0.01182	0.01182	0.01187	0.01176	0.01167

Table 2
Stress tensor Π vs the rarefaction parameter δ and concentration C_0, hard sphere model

δ	Π								
	Ne–Ar			He–Ar			He–Xe		
	$C_0 = 0.1$	0.5	0.9	0.1	0.5	0.9	0.1	0.5	0.9
0.01	0.2786	0.2757	0.2778	0.2732	0.2484	0.2471	0.2701	0.2292	0.2019
0.1	0.2601	0.2576	0.2594	0.2555	0.2335	0.2324	0.2527	0.2163	0.1919
1.0	0.1689	0.1675	0.1685	0.1668	0.1566	0.1562	0.1655	0.1482	0.1360
10.0	0.04150	0.04139	0.04147	0.04137	0.04065	0.04062	0.04128	0.03999	0.03898
40.0	0.01189	0.01188	0.01189	0.01188	0.01182	0.01181	0.01187	0.01177	0.01167

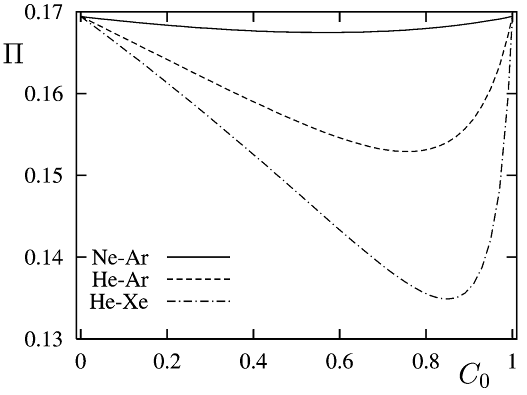

Fig. 1. Stress tensor Π vs concentration C_0 at $\delta = 1$, realistic potential.

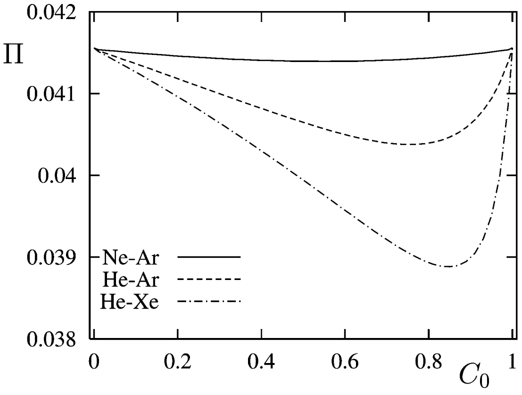

Fig. 2. Stress tensor Π vs concentration C_0 at $\delta = 10$, realistic potential.

Table 3
Comparison of numerical values of the stress tensor Π with the analytical expression (12) for $C_0 = 0.5$, realistic potential

δ	Π					
	Ne–Ar $\sigma_P = 1.040$		He–Ar $\sigma_P = 1.150$		He–Xe $\sigma_P = 1.259$	
	Numerical result	Eq. (12)	Numerical result	Eq. (12)	Numerical result	Eq. (12)
0.1	0.2576	0.2294	0.2336	0.2083	0.2162	0.1910
0.2	0.2414	0.2193	0.2201	0.2000	0.2045	0.1840
0.4	0.2160	0.2016	0.1987	0.1852	0.1856	0.1714
1.0	0.1675	0.1623	0.1566	0.1515	0.1480	0.1421
2.0	0.1239	0.1225	0.1177	0.1163	0.1125	0.1107
4.0	0.08242	0.08224	0.07957	0.07937	0.07700	0.07671
10.0	0.04139	0.04139	0.04065	0.04065	0.03994	0.03994
20.0	0.02264	0.02264	0.02242	0.02242	0.02220	0.02220
40.0	0.01188	0.01188	0.01182	0.01182	0.01176	0.01176

the dimensional stress tensor P_{xy} is related to the reduced one Π via the mean molecular velocity v_0, which depends on the molar concentration monotonously. Thus, the dependence of P_{xy} on the concentration is also monotone even if Π has the minimum at $C_0 = 0.85$.

(iii) The deviation of the stress tensor Π from its value for a single gas is stronger for a mixture with the larger ratio of the molecular mass ratio. For the mixture He–Xe the deviation reaches 25%, while for the mixture Ne–Ar the deviation does not exceed 1.5%.

(iv) The stress tensor is weakly sensitive to the intermolecular interaction law. The difference between the values of Π corresponding to the realistic potential and those for the hard spheres does not exceed 0.5%.

Table 4
Velocity of each species near the plate $y = 1/2$ for the mixture He–Xe, realistic potential

δ	$C_0 = 0.1$		$C_0 = 0.5$		$C_0 = 0.9$	
	u_1	u_2	u_1	u_2	u_1	u_2
0.01	0.0088	0.0132	0.0089	0.0121	0.0104	0.0102
0.1	0.0510	0.0719	0.0517	0.0667	0.0589	0.0579
1.0	0.2049	0.2499	0.2050	0.2382	0.2208	0.2179
10.0	0.4238	0.4401	0.4204	0.4331	0.4243	0.4225
40.0	0.4793	0.4828	0.4775	0.4802	0.4776	0.4767

A comparison of the numerical values of the stress tensor Π with the analytical solution (12) based on the slip boundary condition is performed in Table 3. The data on the slip coefficient σ_{P} are available in the paper [15] for all mixtures considered here. It can be seen that for $\delta \geqslant 10$ the solution (12) coincides with the numerical solution of the kinetic equation within the numerical accuracy. Note, such an agreement can be reached if one uses the correctly calculated slip coefficient, which depends on the composition of mixture. In the range $1 \leqslant \delta < 10$ the expression (12) provides a reasonable approximation, while for $\delta < 1$ the disagreement between Eq. (12) and the exact solution becomes significant. Thus, Table 1 and Eq. (12) cover the whole range of the gas rarefaction δ.

The velocity profiles of each species, i.e. u_1 and u_2, are practically linear functions of the y coordinate. So, it is enough to analyze just the velocity near the upper plate, i.e. at $y = 1/2$. In this point the velocities vary from 0 to 1/2 when the rarefaction parameter δ varies from 0 to ∞. The values of each velocity near the upper plate ($y = 1/2$) for the mixture He–Xe are given in Table 4 for the realistic potential. It can be seen that at $C_0 = 0.1$ and 0.5 the lighter species has a smaller velocity. At $C_0 = 0.9$ the velocities of both species are practically the same.

6. Conclusions

The stress tensor and velocity profiles in the plane Couette flow of binary mixtures were calculated in the whole range of the gas rarefaction. It was found that such type of the flow is weakly affected by the model of the intermolecular interaction. This is consequence of the fact that the Couette flow is determined just by the viscous coefficient. Using any intermolecular potential it is possible to fit its parameters to one transport coefficient. In the case of the Couette flow one can fit the potential parameters to the viscous coefficient and then one obtains the reliable results in the whole range of the gas rarefaction. For large values of the gas rarefaction, i.e. $\delta \geqslant 10$, the analytical solution of the Navier–Stokes equation with the slip boundary condition can be successfully applied if the corresponding slip coefficient is used. Note, the slip coefficient significantly depends on the mixture composition.

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